I. (15 pts) Find the length of the following curve:

$$y = x^4 + \frac{1}{32x^2} \qquad 1 \le x \le 2$$

$$y' = 4x^3 - \frac{1}{32}\left(\frac{x}{x^3}\right)$$

$$y' = 4x^3 - \frac{1}{16x^3}$$

$$L = \int_1^2 \sqrt{1 + y'^2}\, dx$$

$$y'^2 = 16x^6 - \frac{1}{2} + \frac{1}{(16x^3)^2}$$

$$1 + y'^2 = 16x^6 + \frac{1}{2} + \frac{1}{(16x^3)^2} = \left(4x^3 + \frac{1}{16x^3}\right)^2$$

$$\sqrt{1+y'^2} = 4x^3 + \frac{1}{16x^3}$$

$$\therefore L = \int_1^2 \left(4x^3 + \frac{1}{16}x^{-3}\right) dx$$

$$L = x^4 - \frac{1}{32x^2}\Big]_1^2 = 16 - \frac{1}{128} - 1 + \frac{1}{32} = \cancel{\frac{483}{32}} \text{ (u.l}$$

$$= \frac{1923}{128}$$

II. (10 pts) Find the length of the following curve:

$$x = (y+1)^{\frac{2}{3}} \qquad -1 \le y \le 0$$

y	x
-1	0
0	1

$x^{\frac{3}{2}} = y + 1 \Rightarrow y = x^{\frac{3}{2}} - 1$

$\Rightarrow y' = \frac{3}{2} x^{\frac{1}{2}}$

$\Rightarrow y'^2 = \frac{9}{4} x$

$\Rightarrow 1 + y'^2 = 1 + \frac{9}{4} x$

$$\Rightarrow L = \int_0^1 \sqrt{1 + y'^2}\, dx$$

$$L = \int_0^1 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$L = \frac{4}{9}\left(\frac{2}{3}\right)\left(1 + \frac{9}{4}x\right)\sqrt{1 + \frac{9}{4}x}\ \Big]_0^1$$

$$L = \frac{8}{27}\left[\left(1 + \frac{9}{4}\right)\sqrt{1 + \frac{9}{4}} - 1\right]$$

$$L = \cancel{\frac{26}{27}\left(\frac{\sqrt{13}}{2} - 1\right)} \qquad (u.L)$$

$$= \frac{13\sqrt{13}}{27} - \frac{8}{27}$$

III. (15 pts) Find the length of the parametric curve provided below:

$x = 3\sin t$

$y = 3\cos t \qquad 0 \le t \le 2\pi$

$x' = 3\cos t \implies x'^2 = 9\cos^2 t$

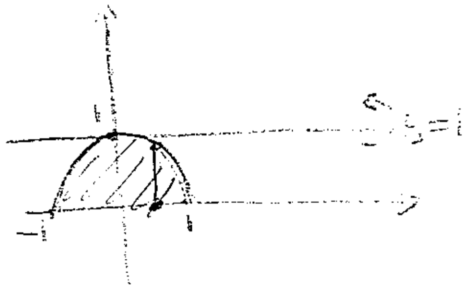
$y' = -3\sin t \implies y'^2 = 9\sin^2 t$

$x'^2 + y'^2 = 9$

$L = 4\int_0^{\frac{\pi}{2}} \sqrt{9}\, dt$

$L = 12t\Big]_0^{\frac{\pi}{2}} = 6\pi$.

IV. (15 pts) Find the volume of the solid generated by rotating the region $0 \le y \le 1 - x^2$ about the line $y = 1$.



$$\cancel{V = \pi \int_{-1}^{1} (1-(1-x^2))^2 \, dx}$$

$$\cancel{V = 2\pi \int_0^1 x^4 \, dx}$$

$$\cancel{V = 2\pi \left(\frac{x^5}{5}\right)_0^1 = \frac{2\pi}{5}} \quad \cancel{(u.v)}$$

$R_o = 1$, $R_i = 1 - (1 - x^2) = x^2$

$$V = \pi \int_{-1}^{1} [1^2 - (x^2)^2] \, dx$$

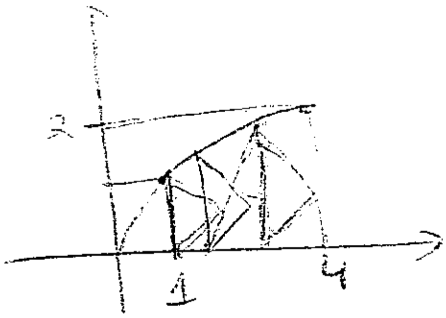
$$= \pi \int_{-1}^{1} (1 - x^4) \, dx$$

$$= \pi \left[x - \frac{x^5}{5}\right]_{-1}^{1} = \frac{8\pi}{5}$$

VI. (10 pts) A solid lies between planes perpendicular to the x-axis at $x = 1 \,\&\, x = 4$. Its cross section perpendicular to the x-axis between these planes is an equilateral triangle with base running from $y = \sqrt{x}$ to the axis. Find the volume of this solid.

$a = \sqrt{x}$ $\quad h = \frac{a\sqrt{3}}{2}$

$h = \frac{\sqrt{x}\cdot\sqrt{3}}{2}$



$$A(x) = \frac{a \times h}{2}$$

$$A(x) = \frac{\sqrt{x}\cdot\sqrt{x}\cdot\sqrt{3}}{4} = \frac{x\sqrt{3}}{4}$$

$$V = \int_1^4 \frac{x\sqrt{3}}{4}\,dx = \frac{\sqrt{3}}{4}\left(\frac{x^2}{2}\right)\Big|_1^4$$

$$= \frac{\sqrt{3}}{8}(16-1)$$

$$= \frac{15\sqrt{3}}{8}.$$ (u.v)

VII. (20 pts – 10 pts each) Given $\vec{u} = 2\vec{i} + 4\vec{j}$ & $\vec{v} = 3\vec{i} - 3\vec{j}$

a) Find $3\vec{u} - 2\vec{v}$

$3\vec{u} = 6\vec{i} + 12\vec{j}$

$-2\vec{v} = -6\vec{i} + 6\vec{j}$

$\Rightarrow 3\vec{u} - 2\vec{v} = 18\vec{j}$

b) Find $\cos\left(\widehat{\vec{u}, \vec{v}}\right)$

$$\cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$\vec{u} \cdot \vec{v} = 6 - 12 = -6$

$\|\vec{u}\| = \sqrt{4+16} = 2\sqrt{5}$

$\|\vec{v}\| = \sqrt{9+9} = 3\sqrt{2}$

$$\Rightarrow \cos(\vec{u}, \vec{v}) = \frac{-6}{6\sqrt{10}} = \frac{-1}{\sqrt{10}}$$